A Unified Model of Network Formation and Network Bargaining

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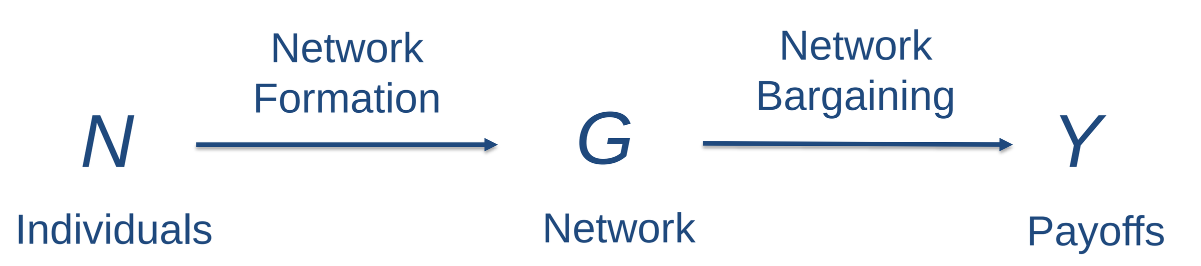
Introduction

- Network Formation
- Network Bargaining
- Previous research: Study these two problems separately
- Our Research: Unify Formation and Bargaining



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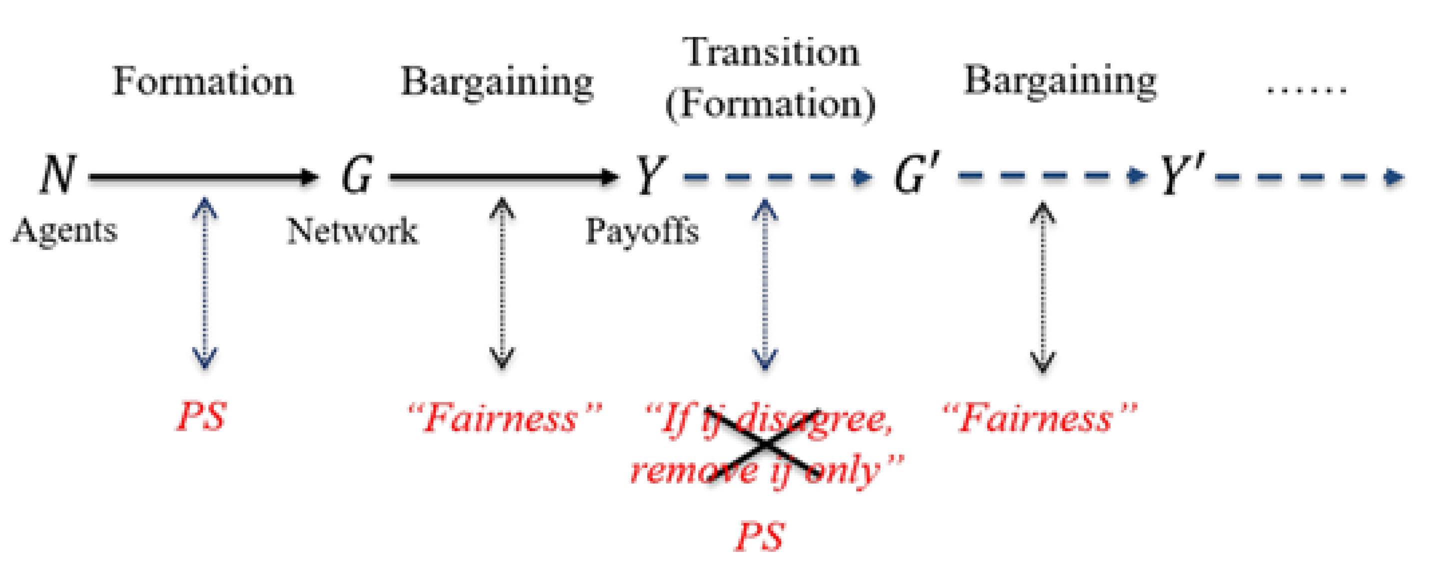
Theory

Consistency:
(1) the networks reached after a counterfactual disagreement are equilibrium networks defined by the network formation solution, and
(2) the network allocation rule at each post-disagreement network is determined by further application of the network bargaining solution.



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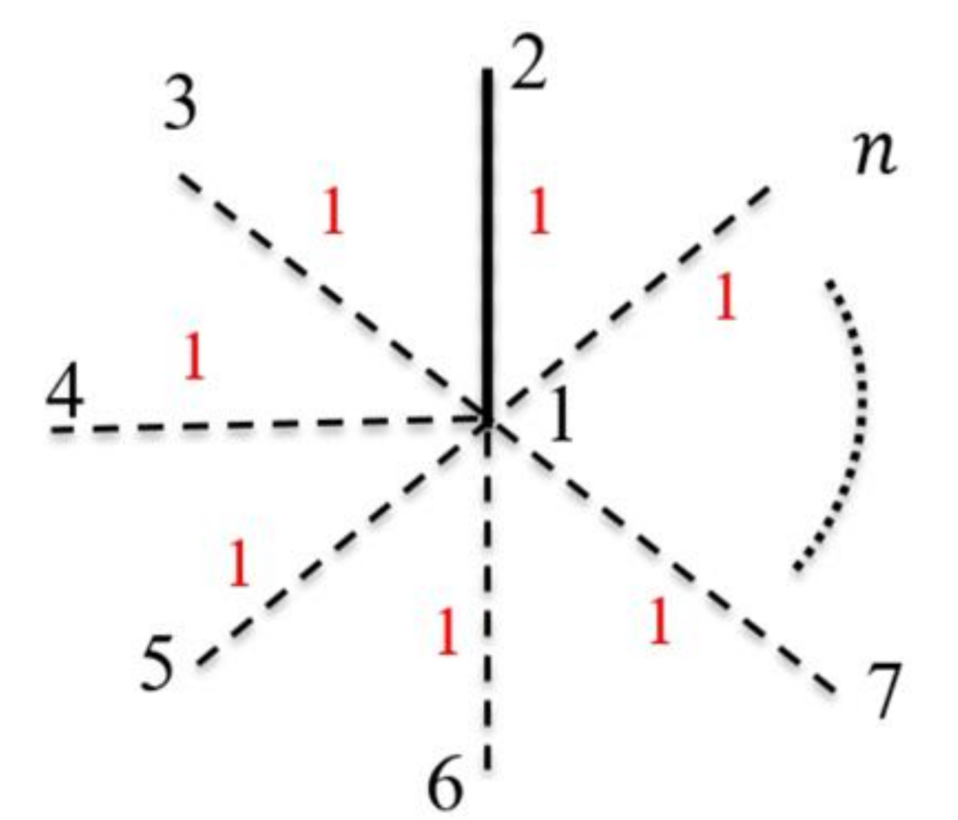
Pairwise stability (PS):
- bilateral consent for link establishment
- robustness to one-link deviations

"Fairness":
- component-wise equal sharing of (nonnegative) net surplus

Key specification:
- if ij disagree (bargaining fails), the link ij is irreversibly removed.

1-Center Example

- n individuals in total
- 1 center with indispensable skill/asset/information for surplus production
- Exactly one link from {1j : j = 2, ..., n}, each of which generates a common value of surplus, namely 1.



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- Utility of the center individual: $X_{c,n} = 1 - \frac{1}{2^{n-1}}$
- Utility of the peripheral individual: $\frac{1}{2^{n-1}}$

What happens if we allow at most 2 links in a network?

- If the surplus of 2-link network, denoted Y_2, is high enough, then the center individual will prefer to form two links for a higher utility.
- Lower bound of Y_2 when 2-link network becomes pairwise stable when there are i individuals in total:

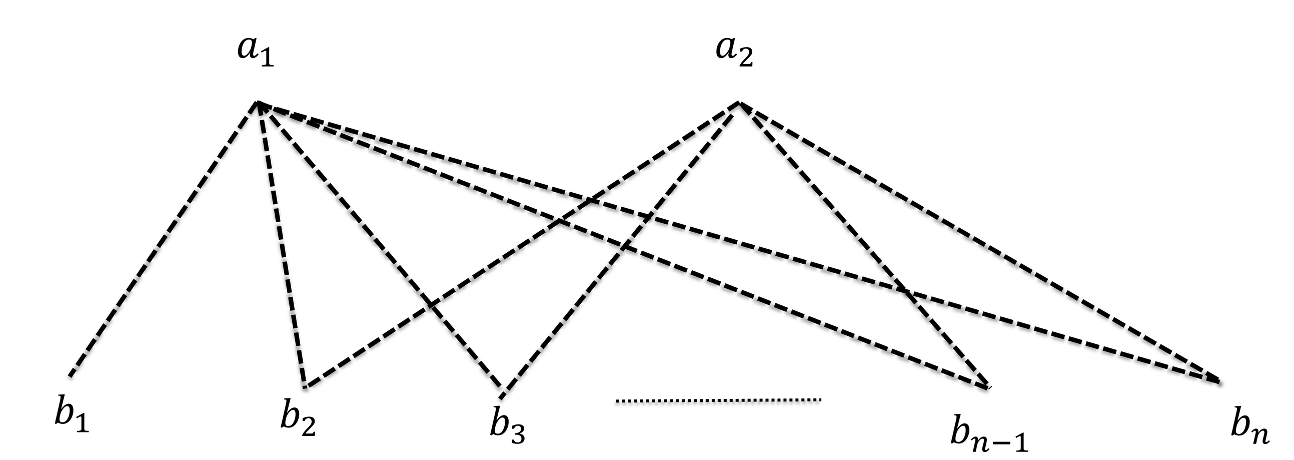
$$Y_2 \geq 1 - \frac{1}{2^{i-1}}$$

What happens if we allow at most k links in a network?

- Depends on the surplus generated by networks with no more than k links
- The center individual always has the power to decide on networks
- Irreversibility of pairwise stable networks
  After a network becomes pairwise stable at certain numbers of individuals versus other networks with fewer links, it will stay pairwise stable when you add more peripheral individuals, until another network structure with even more links take its place
- Allow "skipping"

2-Center Example

- 2 groups of individuals: A and B. A has only two individuals, while there could be many members in B.
- Links could only be formed between an individual in A and an induvial in group B (bipartite graph)
- At most 1 link can be formed for each individual
- Each link generates a surplus of 1



What we have found:

- Center individuals will always choose to connect with an individual that has only one option whenever possible
- General formulas of center individuals' utilities:

$$X_{m,n} = 1 + \left(\frac{1}{2}\right)^{n-m} (X_{m,m} - 1)$$

$$Y_{m,n} = 1 - \left(\frac{1}{2}\right)^{m} - \sum_{k=1}^{m} \frac{1}{n-(m-k)} \left(\frac{1}{2}\right)^{n-m+2k} (1 - X_{m-k,m-k})$$

$$X_{m,m} = \begin{cases} 1 - \left(\frac{1}{2}\right)^{m} - \sum_{j=1}^{m} \frac{1}{m-j+1} \left(\frac{1}{2}\right)^{2(m-j+1)} (1 - X_{j-1,j-1}), & x > 1 \\ \frac{1}{8}, & x = 1 \end{cases}$$

Next Step

- Describe a recursive algorithm which can be used to compute the utility of individuals in 2-center cases
- Extend the algorithm to more general cases
- Construct it on programming software and verify our previous math results

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