Numerical Study of Novel Reconstruction Artifacts from Limited Sonar Tomographic Data

Konstantinos Tsingas[1], Eric T. Quinto[2]

[1]Department of Mathematics, University of Pennsylvania

[2]Department of Mathematics, Tufts University

Introduction

- Two-dimensional sonar tomography uses indirect sound wave data to image the structural boundaries of an underwater object by inverting the circular **Radon transform** over a set of object density data.
- For a **limited data set**, e.g., in the case of object obstruction during imaging, visual artifacts arise from points on the boundary of the object due to the ill-posedness of the inverse integral operator, which makes it difficult to image the object.[1]
- Yet, there exist other physical settings in which morphological changes to the limited data set could incur noisy artifacts that have not yet been explained microlocally,
- We thus focus on finding, characterizing, and reducing novel reconstruction artifacts from limited 2-D sonar data; this will inform the field of tomography in a microlocal context and could have implications for modern inverse imaging technologies.

Limited Data Set Generation

- Consider the ocean $y > 0$ and a source of sound waves as a transceiver on the ocean surface at a point $(x, 0)$, where x is the transceiver center.
- The time distance to an object is the radius r of the circular wave that is reflected by the object. This admits a set $C(x, r)$ of concentric circular waves transmitted at $(x, 0)$ for all centers and radii $x, r \in \mathbb{R}$.
- We let the piecewise smooth object reflectivity function $f(x, y)$ determine the points at which the waves are reflected, namely the boundaries of the object:

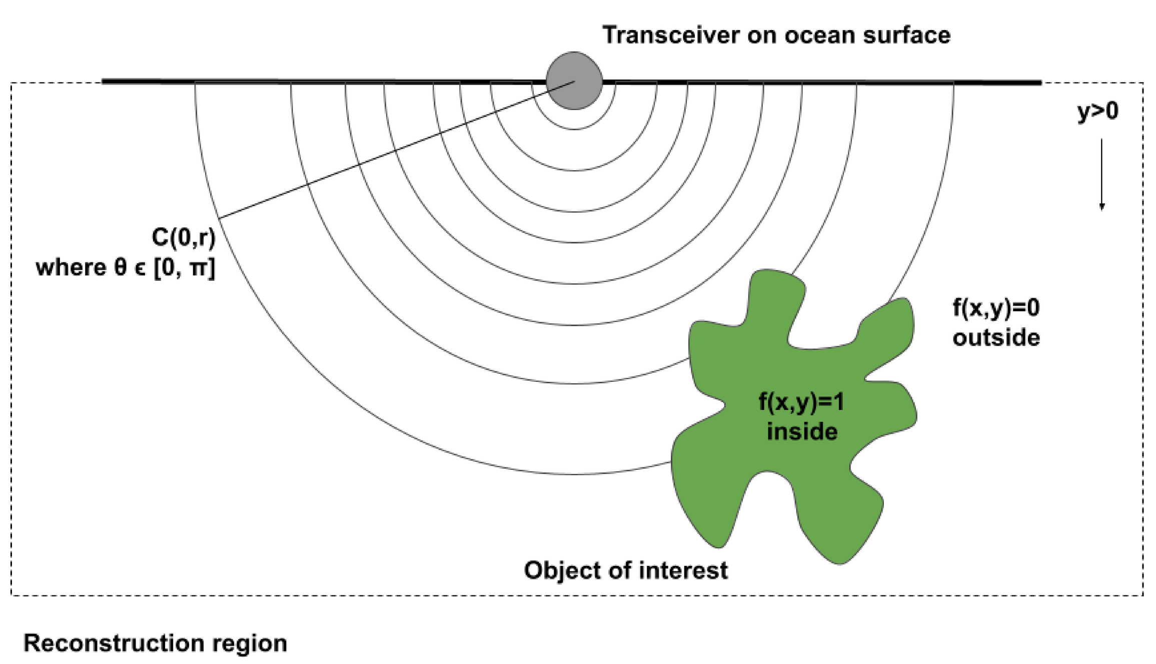


Figure 1: Graphical representation for transceivers with centers (x,0).

- Thus, the wave intensity detected by the transceiver is approximated by the **Sonar transform**, defined parametrically[2]:

$$Rf(x,r) = \int_{C(x,r)} f(x,y)ds = \int_{\theta=0}^{\pi} f(x + r\cos\theta, r\sin\theta)d\theta.$$

- Only a compact subset of $C(x, r)$ is considered such that $(x, r) \in [-T, T] \times [0, R]$, imposing a closed and bounded reconstruction region around the object. This mimics the canonical limited data case.

Object Reconstruction

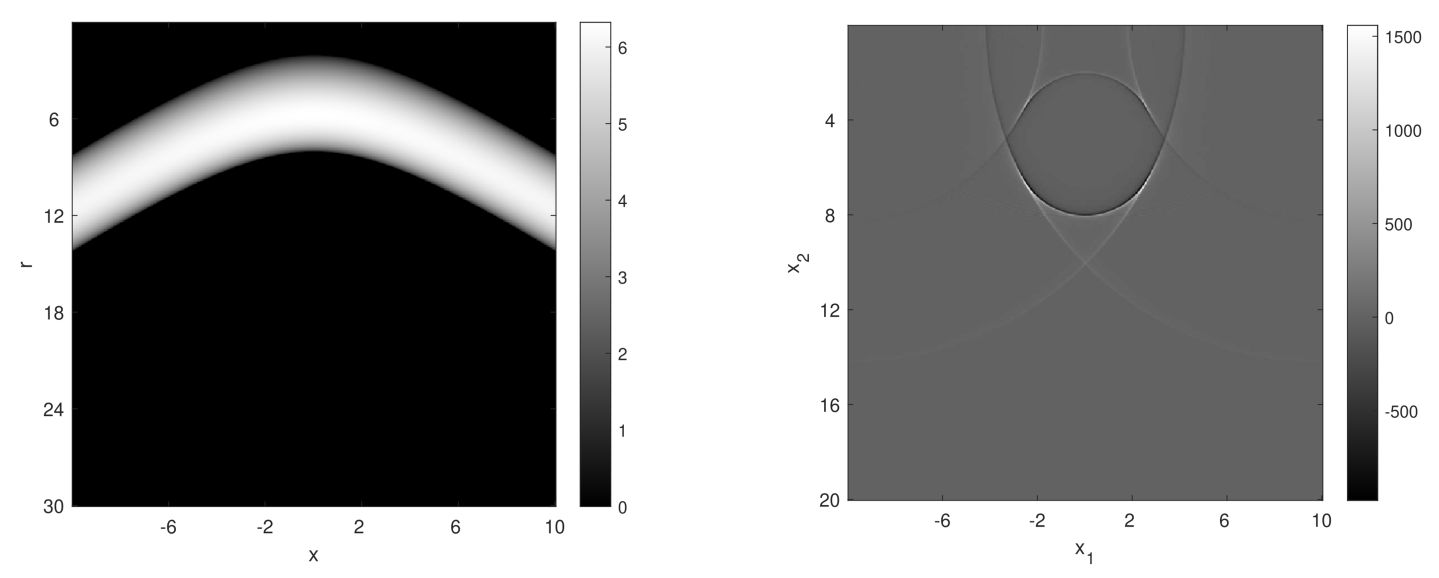
- Consider every point $\bar{x} = (x_1, x_2)$ in the limited reconstruction region and define the **Sonar back-projection** as the inverse operator that recovers the object's properties $f(x, y)$ for all such points:

$$R^*g(\bar{x}) = \int_{x=-T}^{T} g(x, \sqrt{(x_1 - x)^2 + \bar{x}^2})dx.$$

- We thus represent $R^*Rf(\bar{x})$ as the distribution of values of $f(x, y)$ over the entire reconstruction region, imaging the object with visual noise.
- It has been shown that reconstruction artifacts occur when circles $C(x, r)$ that have $(x, r) \in bd(Rf(x, r))$ are tangent to singular points on $bd(f(x, y))$; the artifacts visually spread along such circles.[1,3]
- Similarly, a point on $bd(f(x, y))$ will only be visible if there are any circles tangent to that point.

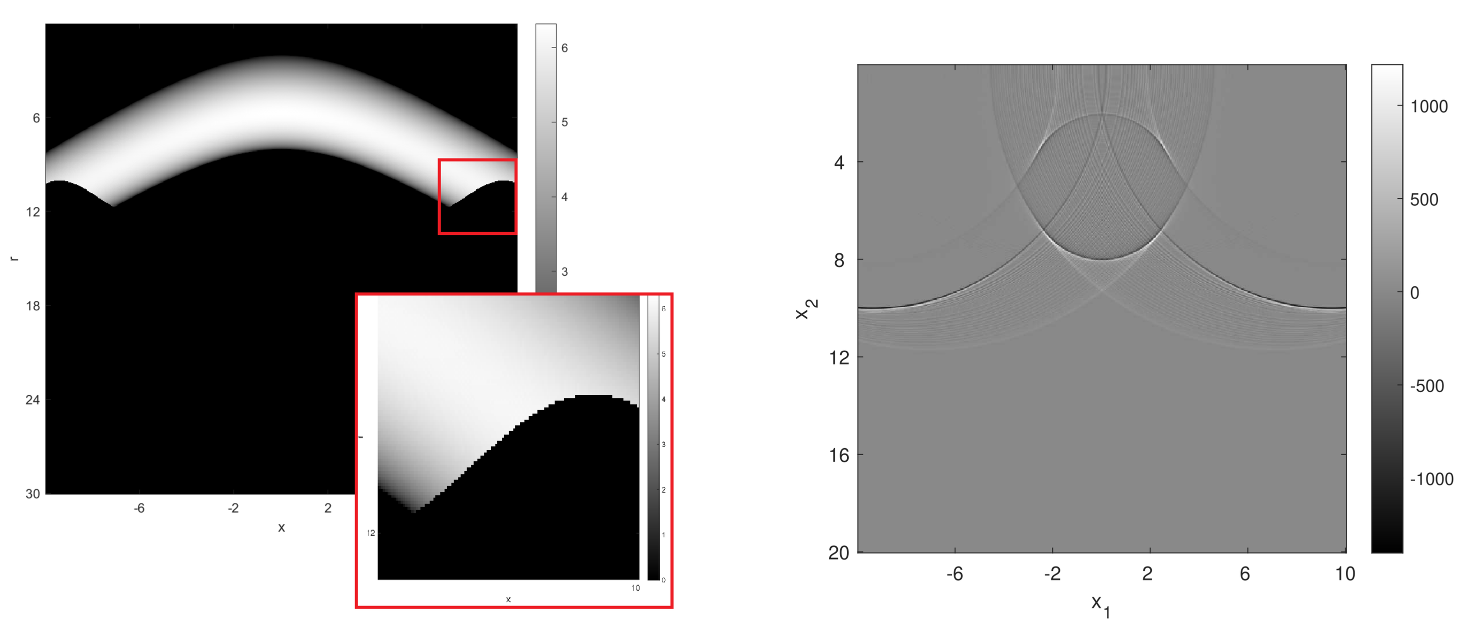
Numerical Methods and Experiments

- To simulate the limited data set, we tabulate $R^*Rf(\bar{x})$ into a data matrix and approximate integrals using trapezoidal rule.
- Since f is a piecewise smooth function, we apply a 3-point second order central difference D to accentuate boundaries and visual artifacts.



(a) (b)

Figure 2: (a) Data set for a disk centered at (0,5) with radius 3 and (b) its subsequent filtered object reconstruction, generated using $R^*DRf(\bar{x})$.



(a) (b)

Figure 3: (a) Data set for a disk truncated by $cos(x)$ with non-smooth points on $bd(Rf(x, r))$ and (b) its image with semi-concentric artifacts independent of the disk.

Artifact Reduction

- A common artifact reduction method is multiplying the data with a smooth cutoff function ψ with compact support.[3] Yet, this conflates the data and does not localize to $bd(Rf(x, r))$.
- So, we convolve over a scanning window on the data matrix and smooth out jump discontinuities on $bd(Rf(x, r))$ based on some tolerance.
- For some data set F and a function ϕ symmetric about 0 such that $\sum_{i=-n}^{n} \phi(i) = 1$, the **n-point discrete convolution** is defined as:

$$C_\phi g(x,r) := (g * \phi)(x,r) = \sum_{j=-n}^{n} g(x, r-j)\phi(j).$$

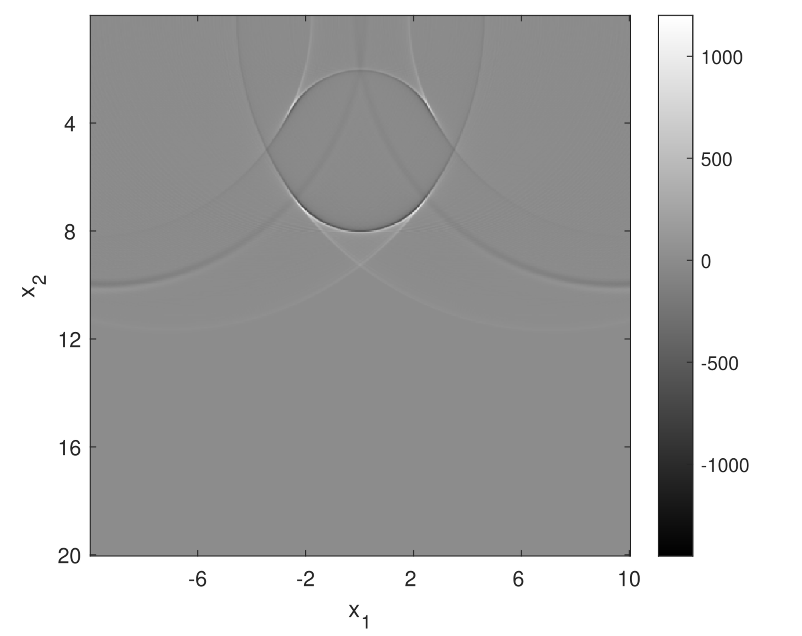


Figure 4: Disk smoothed by 7-point convolution $R^*DC_\phi Rf(\bar{x})$ with dissipated artifacts.

Conclusions and Future Directions

- We find novel reconstruction artifacts that occur from non-smooth points on $bd(Rf(x, r))$ independent of $bd(f(x, y))$. We develop a convolution algorithm that minimizes these and any canonical artifacts.
- We aim to apply this research to microlocal theory, other limited data scenarios, and other general imaging problems.

References

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ktsingas@sas.upenn.edu