Learning Continuous Chaotic Attractors with a Reservoir Computer

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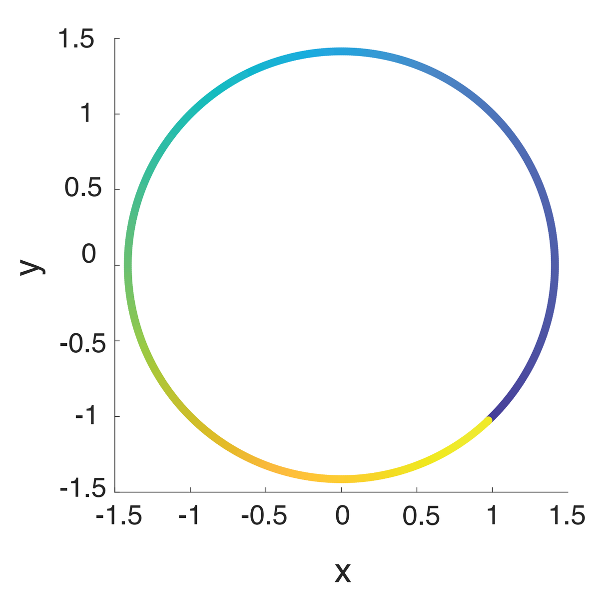
Motivation

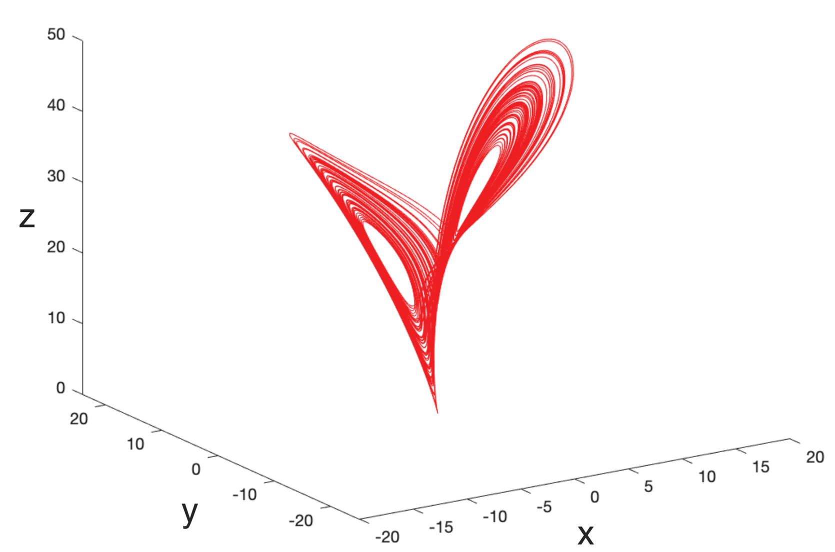
Neural networks have the ability to learn and abstract information from only a few examples shown, with the implications being that we can improve artificial intelligence in many fields, such as self-driving cars, while allowing us a deeper understanding of how these computers make these computations.

Methods



Limit cycle equation:

$$\dot{\mathbf{x}}(t) = 10\mathbf{x}(t)(2 - \mathbf{x}^2(t) - \mathbf{y}^2(t)) - 10\mathbf{y}(t)$$
$$\dot{\mathbf{y}}(t) = 10\mathbf{x}(t)(2 - \mathbf{x}^2(t) - \mathbf{y}^2(t)) + 10\mathbf{x}(t)$$

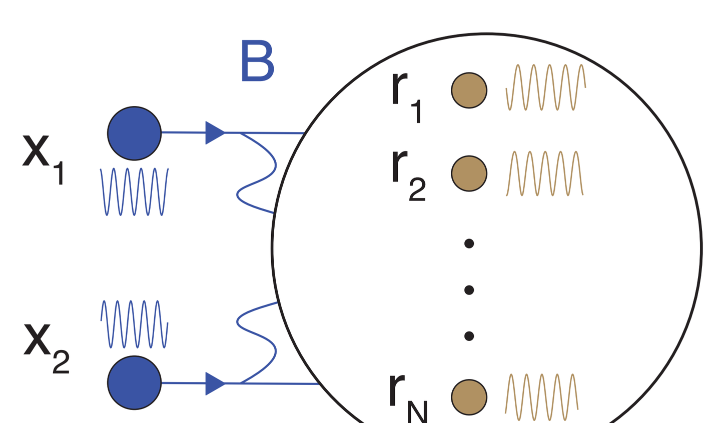


Lorenz attractor equation:

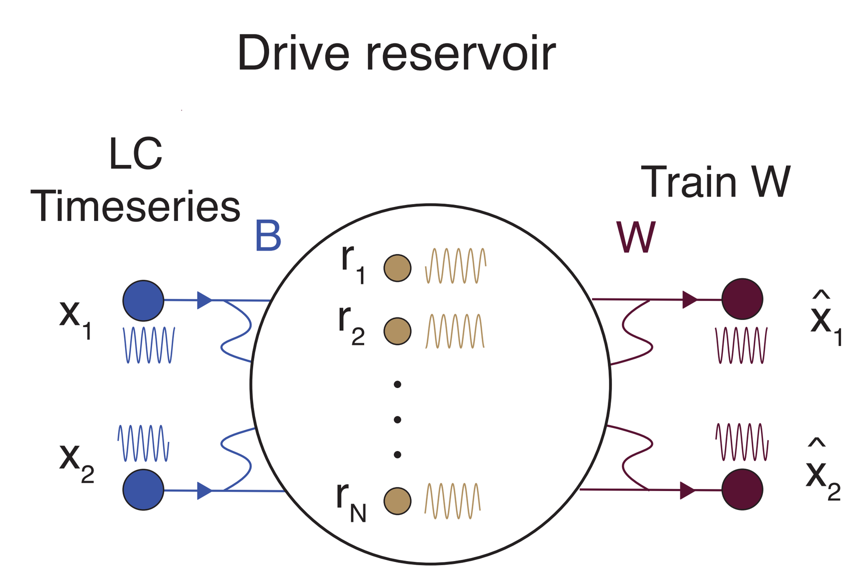
$$\dot{\mathbf{x}}(t) = -10(\mathbf{x}(t) - \mathbf{y}(t))$$
$$\dot{\mathbf{y}}(t) = 28\mathbf{x}(t) - \mathbf{y}(t) - \mathbf{x}(t)\mathbf{z}(t)$$
$$\dot{\mathbf{z}}(t) = -8/3\mathbf{z}(t) + \mathbf{x}(t)\mathbf{y}(t)$$

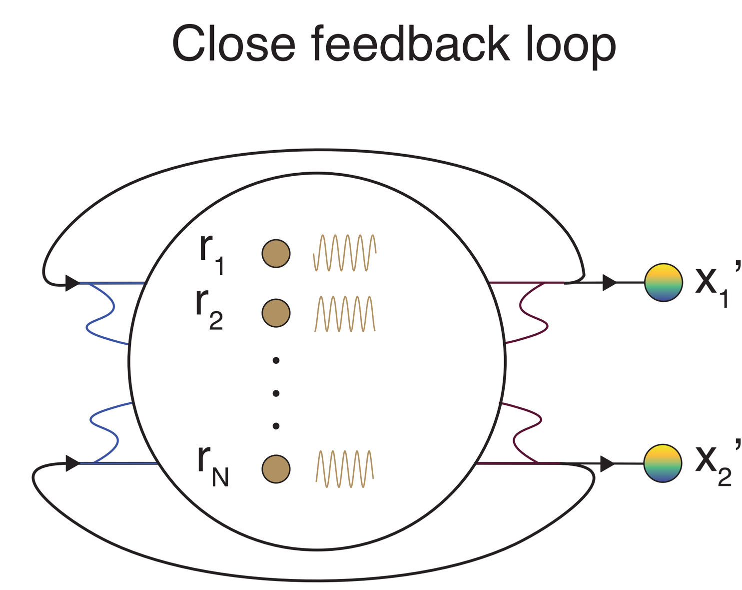
Reservoir computer equation:

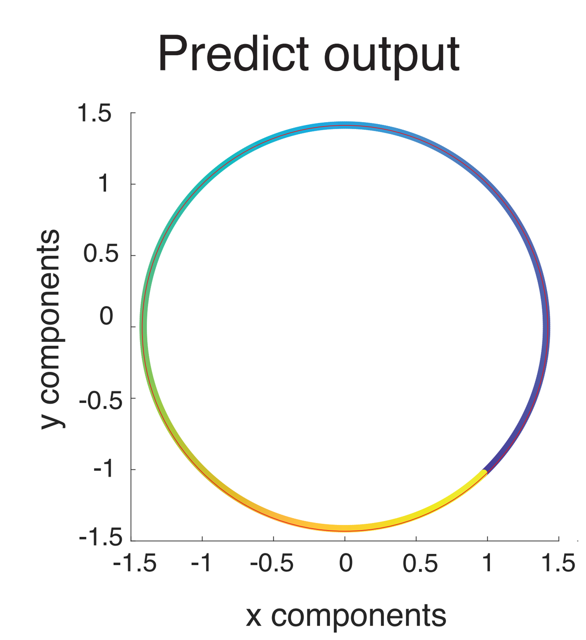
$$\dot{\mathbf{r}}(t) = \gamma \tanh(A\mathbf{r}(t) + B\mathbf{x}(t) + d) - \gamma\mathbf{r}(t)$$



Training the RNN





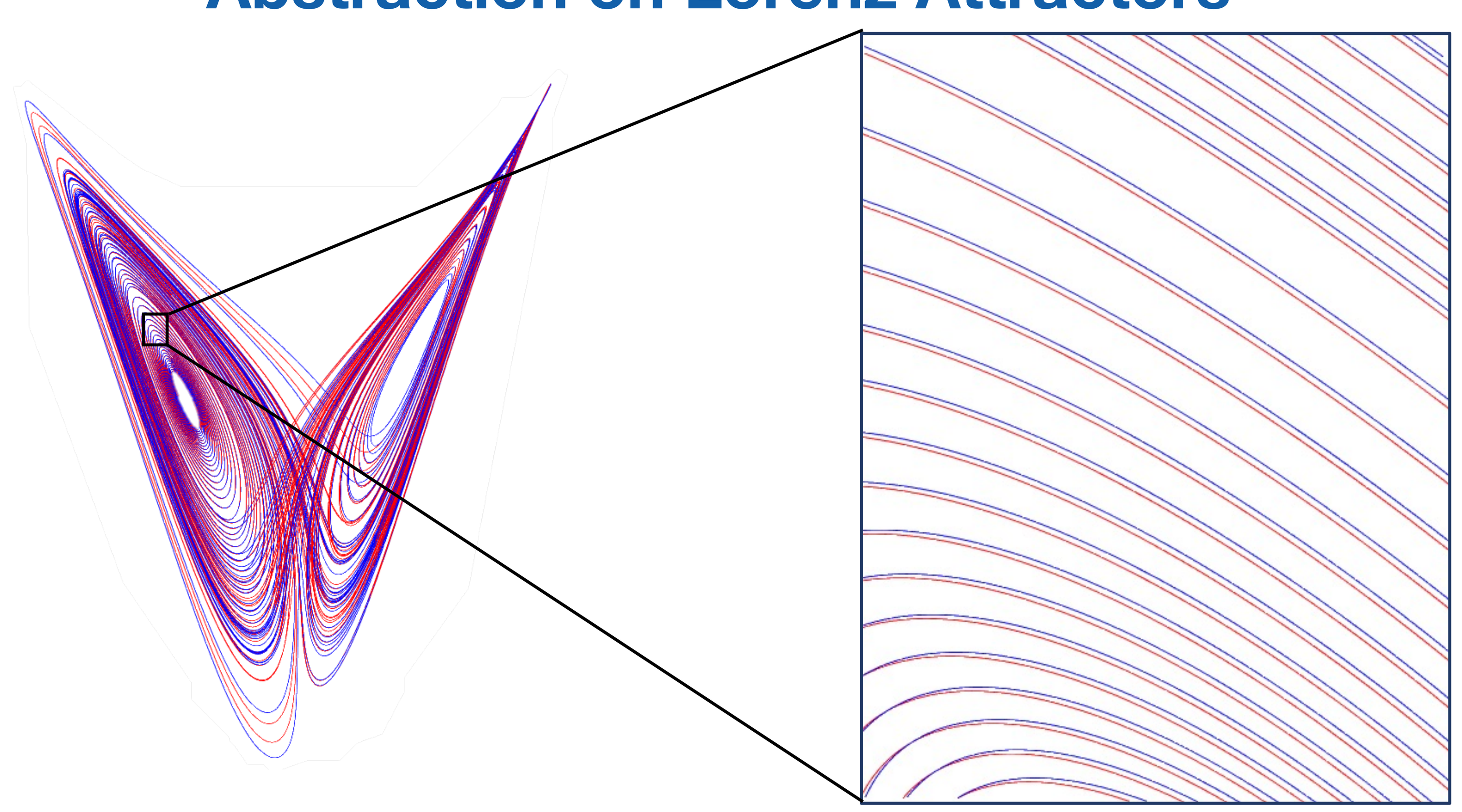


When the feedback loop is closed, the outputs become the new inputs, which drives the RC autonomously.

Autonomous RC equation:

$$\dot{\mathbf{r}}(t) = \gamma \tanh((A + BW)\mathbf{r}(t) + d) - \gamma\mathbf{r}(t)$$

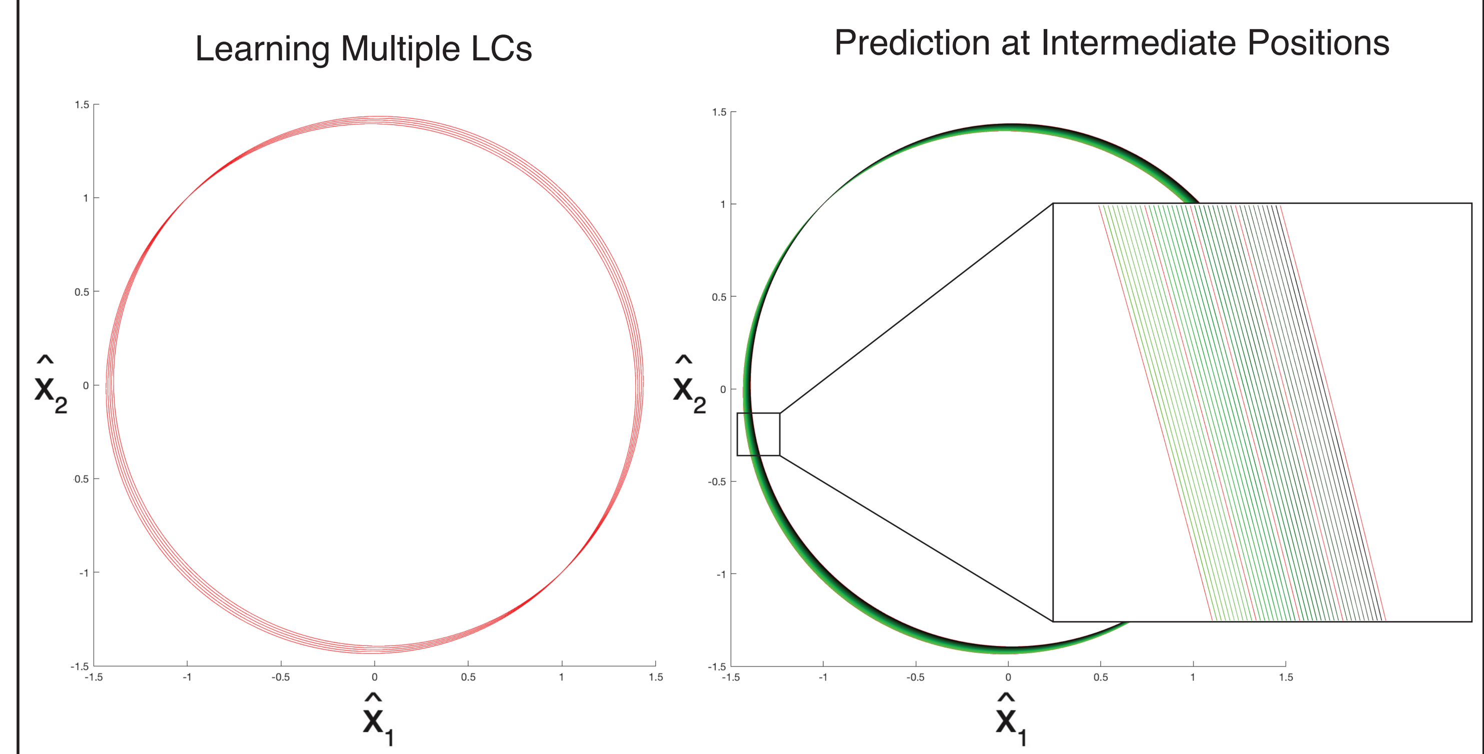
Abstraction on Lorenz Attractors

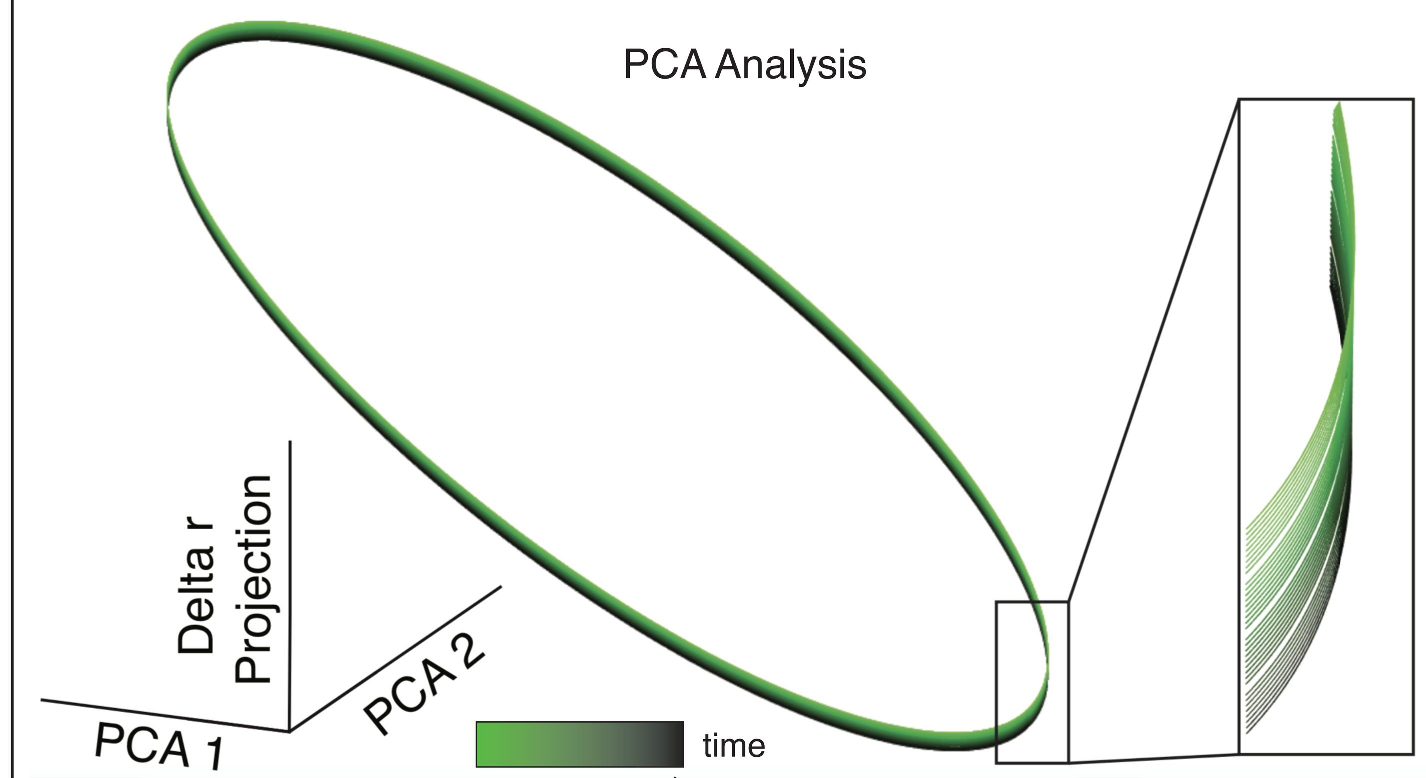


We show the RC multiple discrete examples shifted in one dimension, and the RC will abstract in an additional fourth dimension to create a continuous Lorenz attractor. When calculating the Lyapunov exponents of the RC, a second 0 LE appears, indicating successful abstraction.

This method of abstraction could be applied to other attractors as well. Humans perform abstraction when making inferences or predictions, so an RNN performing abstractions could provide insight into the mechanisms humans utilize.

Abstraction on Limit Cycles





We show the RC multiple discrete examples shifted in one dimension, and the RC will abstract in an additional third dimension to create a continuous limit cycle attractor. The PCA analysis highlights the additional dimension, on the delta R axis. When calculating the Lyapunov exponents of the RC, a second 0 LE appears, indicating successful abstraction.

References & Acknowledgements

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