3D Anisotropic Scattering Transformation

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Background and Goal

- The scattering transform is **statistical method** that produces **summary statistics** for a field
- Created by iteratively convolving a wavelet with a target field and taking the modulus of that field
- Captures **non-Gaussian information** and **local interactions** in a small number of coefficients
- Significantly more computationally efficient than a Convolutional Neural Network
- Our goal is to develop a **3D anisotropic scattering transform** to capture directional and local information in 3D

Mathematics

- We took a 2D directional wavelet defined on R^2 and through a unitary diffeomorphic inverse stereographic projection to send it to a sphere S^2
- We **mapped the 2D directional properties onto a 3D surface** (following Wiaux et al.'s procedure)
- In some cases we convolved this by spherical harmonics and in all cases multiplied by a radial Gaussian, which created a 3D directional wavelet
- We applied this into the Kymatio python package

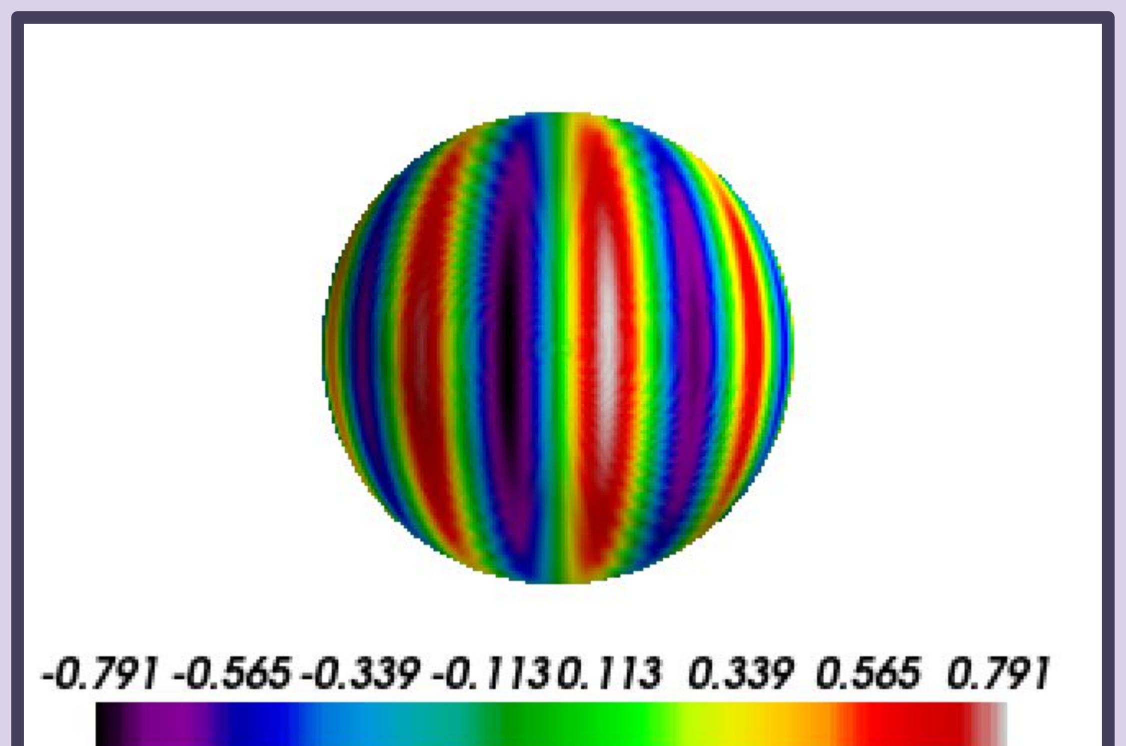

Figure 1 - Image of one of our directional wavelets on a sphere after projection. North pole facing observer with red regions positive and blue regions negative. The striped alternation between red and blue demonstrates the directionality.

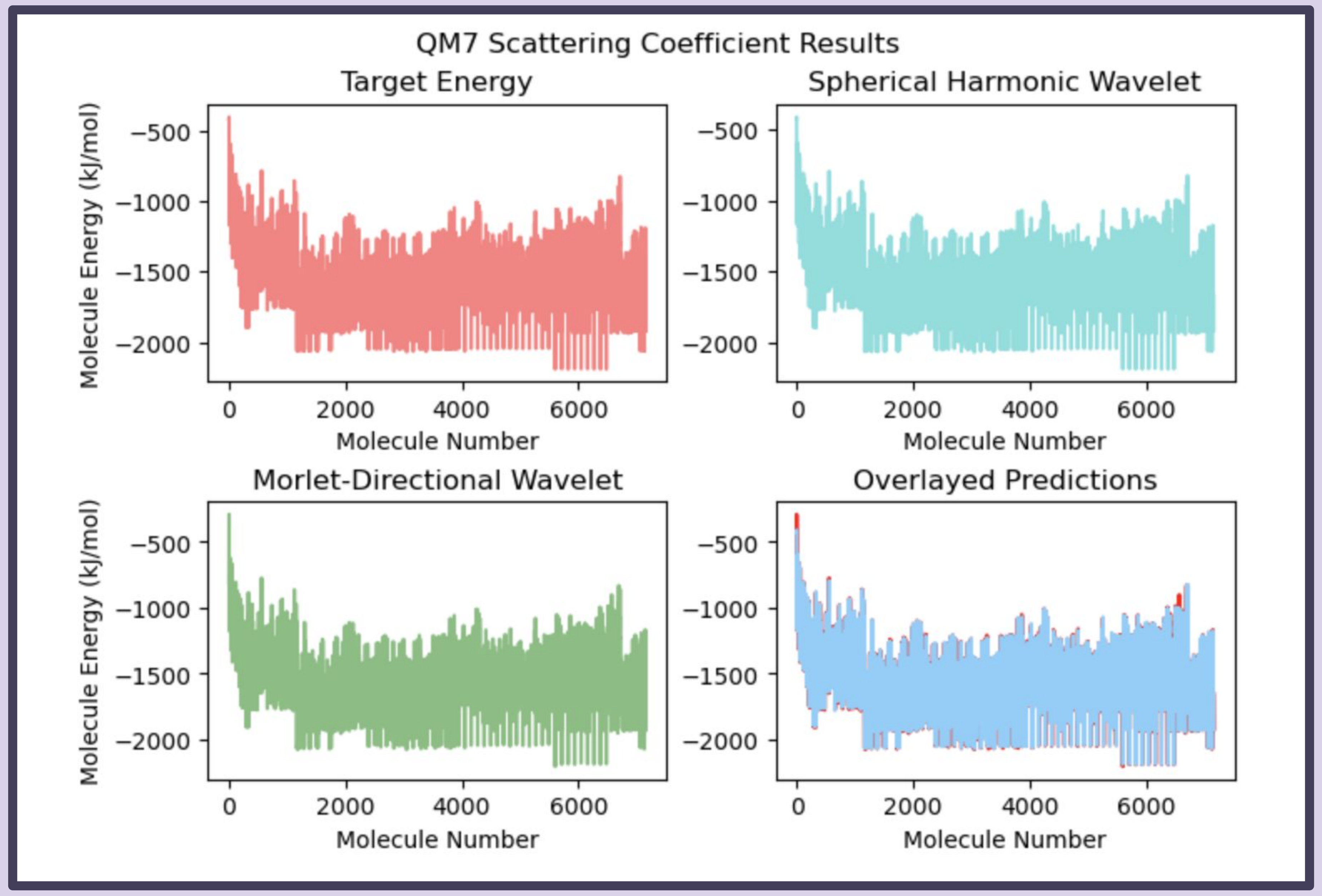

Figure 2 - Estimated molecule energy for each molecule number. Bottom right corner shows a comparison of our wavelets predictions with the existing method's predictions where we see a clear overlap (blue) with very few disagreements (red).

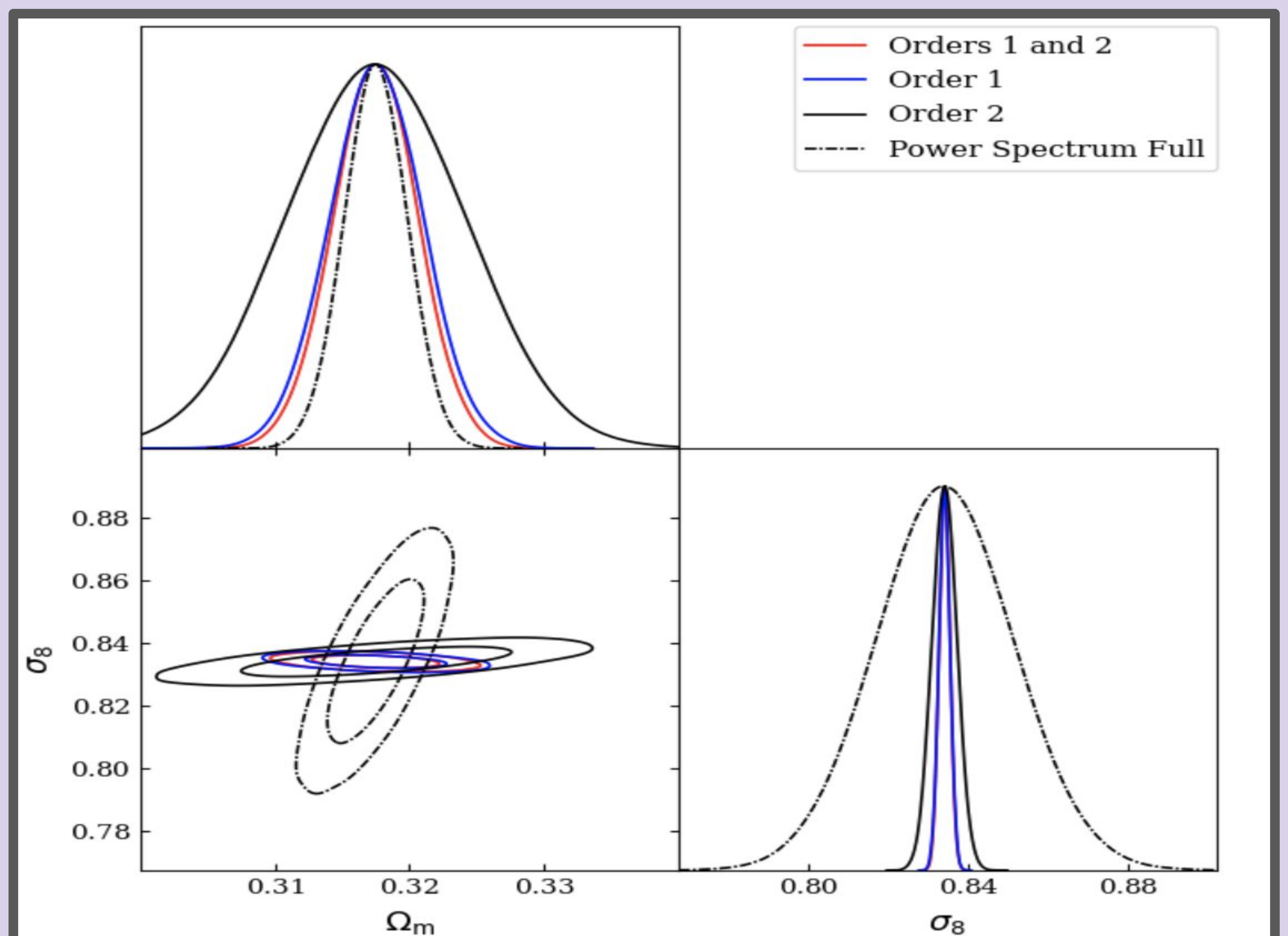

Figure 3 - Fisher forecast of the Om-s8 tension using our wavelet as compared to the power spectrum. The ellipses graphed show one and two sigma confidence for our prediction. We see a strong convergence using our method, demonstrating that our wavelet is capable of providing a strong analysis of the universe's Large Scale Structure as generated by the Quijote simulation.

Results

Quantum Energy Data

- We first applied our wavelet to the QM7 data set of rotationally invariant molecules (quantum chemistry data)
- Our results in Figure 2 demonstrate that our **wavelet performs just as well on non-directional data as currently existing methods** (MAE is on the same order)

Large Scale Structure (LSS) Fisher Analysis

- We apply our wavelet and Kymatio's 3D scattering transform to analyze the Quijote Simulation Large Scale Structure data at redshift 0
- LSS data contains non-directional filaments, so we use our wavelet to test its **ability to capture non-directionality**
- Using our wavelet we run a **Fisher Matrix Analysis** to demonstrate the **s8-Om tension** and constrain these cosmological parameters
- Our wavelet performs **better** than the **power spectrum**
- Further work is needed to better the analysis and compare it to existing Fisher Analysis using scattering transform

Image Generation Using our Wavelet

- Using our scattering coefficients as a target we take a random field and **perturb** it until it **matches the scattering coefficients** of the original field
- This generates fields with similar properties to the original

Acknowledgements

- Penn Undergraduate Research Mentoring Program (PURM) funding
- MJ and SK thank postdocs Marco Gatti and Tanvi Karwal

References

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